

56. where does normal line intersect the curve
 $(1,1)$ we want another point (x,y)

need 2 eqns. 1. $x^2 + 2xy - 3y^2 = 0$

2. Normal line $y = -(x-1)+1$

slope: \perp

$$2x + 2xy' + y \cdot 2 - 6yy' = 0$$

$$2xy' - 6yy' = -2x - 2y$$

$$y'(2x - 6y) = -2x - 2y$$

$$y' = \frac{-2x - 2y}{2x - 6y} \Big|_{(1,1)} = \frac{-4}{-4} = 1$$

slope of
tan

↓

$m_{\perp} = -1$

solve $(x^2 + 2xy - 3y^2 = 0, x) \mid y = -(x-1)+1$

$$x=1 \quad x=3$$

$$y=1 \quad y=-1$$

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57. find
 normals (eqn)
 that are

parallel to $2x + y = 0$

same slope as \uparrow

need a pt. (x,y)

$$m_{\perp} = -2$$

$$y' \leftarrow m_{\text{tan}} = \frac{1}{2}$$

$$y' = \frac{1}{2}$$

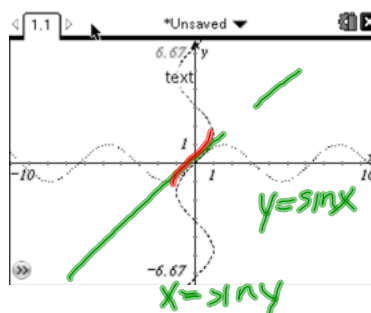
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3.8 Derivatives of inverse trig functions

Derivative of the Arcsine

 $y = \sin^{-1}(x)$ means $x = \sin(y)$

$y = \arcsin(x)$

restrict the range to make $y = \arcsin(x)$ a function

$$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos x = \sqrt{1 - \sin^2 x}$$

$$x = \sin y$$

$$1 = y' \cos y$$

$$y' = \frac{1}{\cos y}$$

$$= \frac{1}{\sqrt{1 - \sin^2 y}}$$

$$= \frac{1}{\sqrt{1 - x^2}}$$

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$$\frac{d}{dx} (\sin^{-1} x^2) = 2x \cdot \frac{1}{\sqrt{1-(x^2)^2}} \quad u = x^2$$

$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \left(\sin^{-1} \frac{\sqrt{x}}{3} \right) = \frac{1}{6} x^{-\frac{1}{2}} \cdot \frac{1}{\sqrt{1 - \left(\frac{\sqrt{x}}{3} \right)^2}} = \frac{1}{6\sqrt{x}\sqrt{1 - \frac{x}{9}}}$$

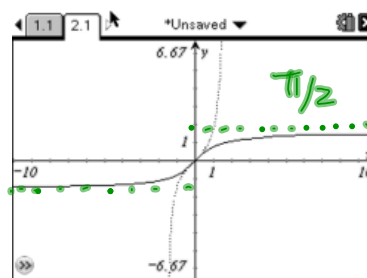
$$\frac{1}{3} \sqrt{x}$$

$$\frac{1}{3} x^{\frac{1}{2}}$$

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Derivative of the Arctangent

$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}$$

 $-\frac{\pi}{2}$
What is the range of $y = \text{Arctan}(x)$?

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A particle moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = \tan^{-1} \sqrt{t}$. What is the velocity of the particle when $t=16$?

$$\left. \frac{dx}{dt} \right|_{t=16}$$

$$x = \tan^{-1}(t^{1/2})$$

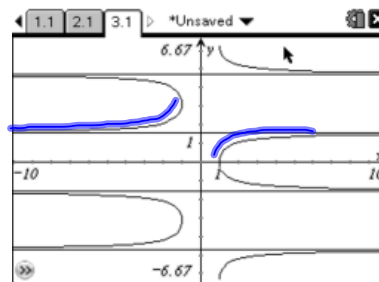
$$\frac{dx}{dt} = \frac{1}{2} t^{-1/2} \cdot \frac{1}{1+(\sqrt{t})^2} = \frac{1}{2\sqrt{t}(1+t)}$$

$$\begin{aligned} x'(16) &= \frac{1}{2 \cdot 4 \cdot 17} \\ &= \frac{1}{136} \end{aligned}$$

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Derivative of the Arcsecant

$$\frac{d}{dx} \sec^{-1} u = \frac{1}{|u| \sqrt{u^2 - 1}} \frac{du}{dx}$$



restrict the range to make $y = \text{Arcsec}(x)$ a function

$$0 \leq \sec^{-1} x \leq \pi$$

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$$\frac{d}{dx} \sec^{-1}(5x^4) = 20x^3 \cdot \frac{1}{|5x^4| \sqrt{(5x^4)^2 - 1}}$$

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Derivatives of the other three

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$\csc^{-1} x = \frac{\pi}{2} - \sec^{-1} x$$

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Derivative of an inverse function

If f and g are inverse functions then $g'(x) = \frac{1}{f'(g(x))}$

$$g'(x) = \frac{1}{f'(y)}$$

$$\{y = g(x)\}$$

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